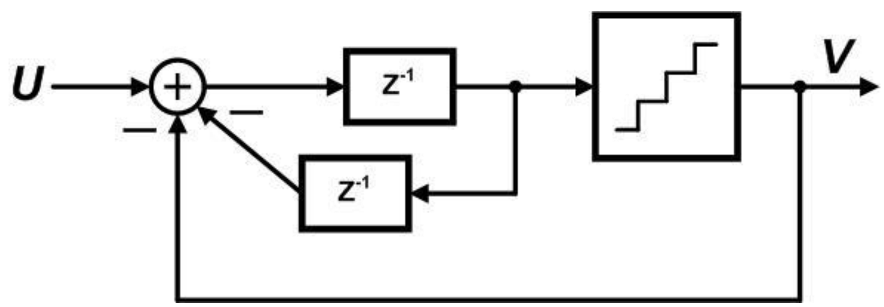
ECE 627 Spring 2010

FINAL EXAMINATION
June 9, 2008
Open book

Name: ____________________________________

1. The signal transfer function (STF) of a delta-sigma modulator is 1, the noise transfer function (NTF) is $(1 - z^{-1})^2$. The LSB voltage of the quantizer is 0.1 V. The input signal is $u(t) = \sin(2\pi f t)$, where $f = 1$ kHz. The clock frequency is 128 kHz.
   a. Find the largest possible difference between $u(n)$ and $v(n)$.
   b. Find the largest change $|y(n) - y(n-1)|$ in the input signal of the quantizer.

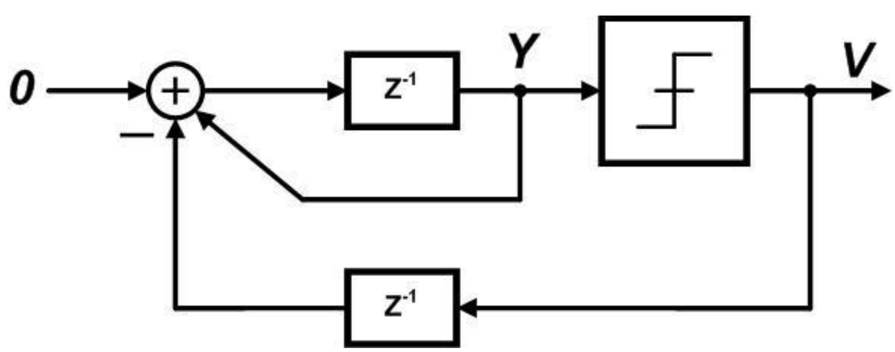
2. Find the STF(z) and NTF(z) for the modulator shown below. Calculate the zeros and poles of both STF and NTF, in terms of z as well as f/f_S.



3. The modulator shown below has zero input at all times. The quantizer input y(n) is zero for both $n = -1$ and $n = -2$. The quantizer characteristics are

$$v(n) = -1 \text{ if } y(n) < 0; \qquad v(n) = +1 \text{ if } y(n) \geq 0$$

Calculate and plot v(n) for n = 0, 1, 2, ..., 10. (Hint: it is periodic)



1. a.

$$u(n) = \sin(2\pi f n/f_S) = \sin(n\pi/64)$$

$$V(z) = U(z) + (1 - 2z^{-1} + z^{-2})E(z)$$

$$v(n) = u(n) + e(n) - 2e(n-1) + e(n-2)$$

$$v(n) - u(n) = e(n) - 2e(n-1) + e(n-2)$$

$$|v(n) - u(n)|_{max} = 4|e(n)|_{max} = 2V_{LSB}$$

$$|v - u| \leq 0.2\ V$$

b.

$$y(n) = v(n) - e(n) = u(n) - 2e(n-1) + e(n-2)$$

$$y(n-1) = u(n-1) - 2e(n-2) + e(n-3)$$

$$y(n) - y(n-1) = u(n) - u(n-1) - 2e(n-1) + 3e(n-2) - e(n-3)$$

$$|y(n) - y(n-1)|_{max} = \sin(\pi/64) + 6|e(n)|_{max}$$

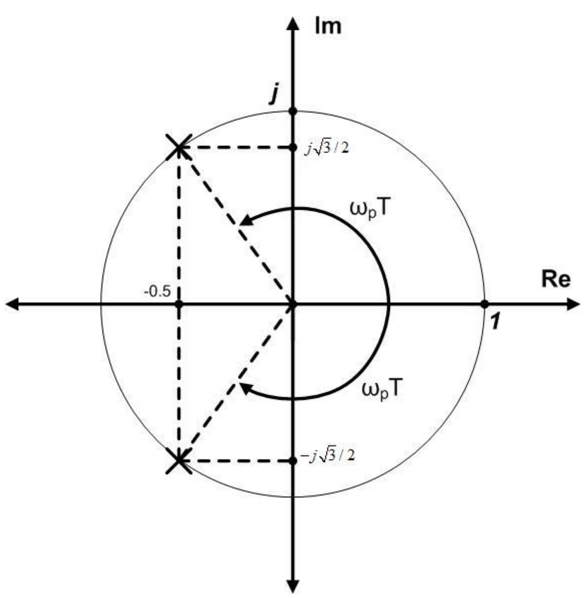
$$\cong 0.0491 + 6(0.5)V$$

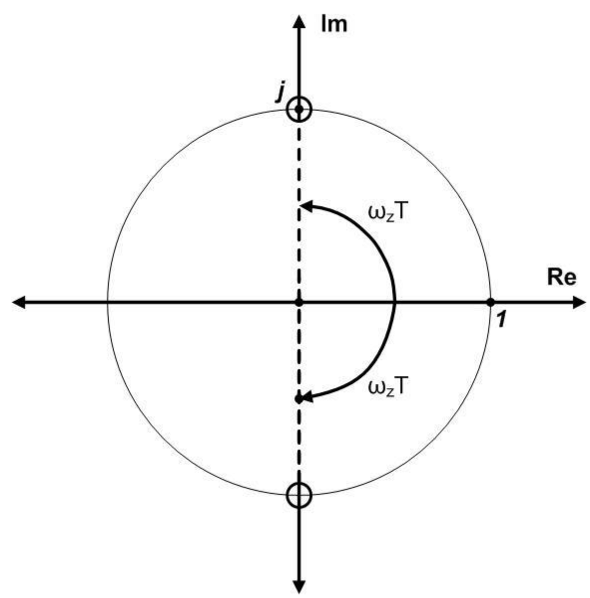
$$|y(n) - y(n-1)| \leq 0.3491\ V$$

2.

$$V = E + z^{-1}[U - V - z^{-1}(V - E)]$$

$$(1 + z^{-1} + z^{-2})V = z^{-1}U + (1 + z^{-2})E$$





Signal Transfer Function

$$\Longrightarrow STF = \frac{z}{z^2 + z + 1}$$

Zeros: $z_z = 0, \infty$

Zeros not on unit circle in f/f_S domain

Poles: $z_p = \frac{-1 \pm j\sqrt{3}}{2}$

$$e^{j\omega_p t} = \frac{-1 \pm j\sqrt{3}}{2}$$

$$\Longrightarrow \frac{j\omega_p}{f_S} = \frac{j2\pi f_p}{f_S} = \ln\left(\frac{-1 \pm j\sqrt{3}}{2}\right) = \pm\frac{j2\pi}{3}$$

$$\Longrightarrow \frac{f_p}{f_S} = \pm\frac{1}{3}$$

Noise Transfer Function

$$\Longrightarrow NTF = \frac{z^2 + 1}{z^2 + z + 1}$$

Zeros: $z_z = \pm j$

$$e^{j\omega_p t} = \pm j$$

$$\Longrightarrow \frac{j\omega_z}{f_S} = \frac{j2\pi f_z}{f_S} = \ln(\pm j) = \pm\frac{j\pi}{2}$$

$$\Longrightarrow \frac{f_z}{f_S} = \pm\frac{1}{4}$$

Poles: $z_p = \frac{-1 \pm j\sqrt{3}}{2}$

$$\Longrightarrow \frac{f_z}{f_S} = \pm\frac{1}{3}$$

3.

$$v(n) = \mathrm{sgn}[y(n)]$$

$$y(n) = y(n-1) - v(n-2)$$

$$y(0) = y(-1) - v(-2) = 0 - 1 = -1$$

$$\Longrightarrow v(0) = -1$$

$$y(1) = y(0) - v(-1) = -1 - 1 = -2$$

$$\Longrightarrow v(1) = -1$$

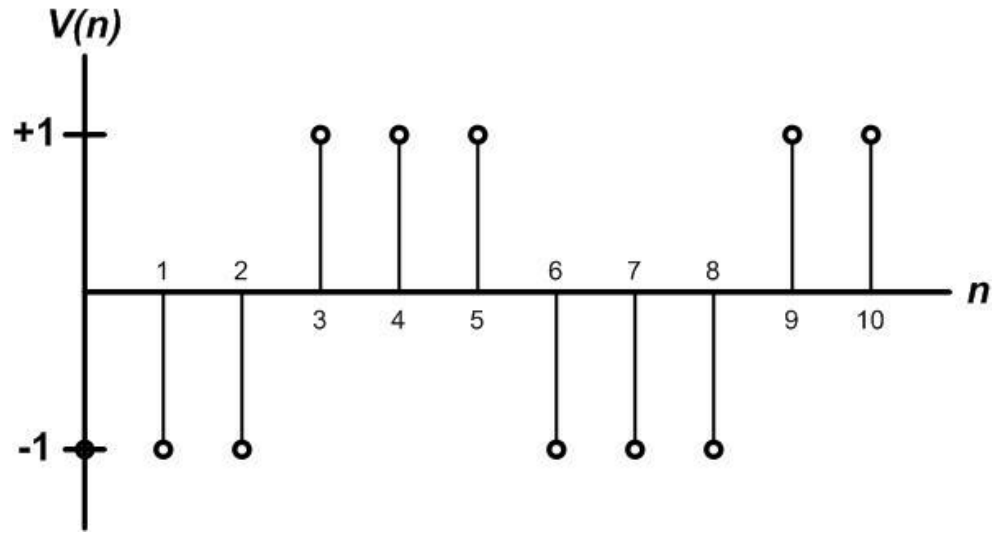
$$y(2) = y(1) - v(0) = -2 - (-1) = -1$$

$$\Longrightarrow v(2) = -1$$

$$y(3) = y(2) - v(1) = -1 - (-1) = 0$$

$$\Longrightarrow v(3) = +1$$

… repeat through n=10


V(n)
+1
-1
1
2
3
4
5
6
7
8
9
10
n